

Short-time Loschmidt gap in dynamical systems with critical chaos

Carl T. West,^{1,3} Tomaz Prosen,² and Tsampikos Kottos^{1,3}

¹Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA

²Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, SI-1000 Ljubljana, Slovenia

³Max-Planck-Institute for Dynamics and Self-Organization, 37073 Göttingen, Germany

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We study the Loschmidt echo $F(t)$ for a class of dynamical systems showing critical chaos. Using a kicked rotor with singular potential as a prototype model, we found that the classical echo shows a gap (initial drop) $1 - F_g$, where F_g scales as $F_g(\alpha, \epsilon, \eta) = f_{cl}(\chi_{cl} \equiv \eta^{3-\alpha}/\epsilon)$; α is the order of singularity of the potential, η is the spread of the initial phase-space density, and ϵ is the perturbation strength. Instead, the quantum echo gap is insensitive to α , described by a scaling law $F_g = f_q(\chi_q = \eta^2/\epsilon)$ which can be captured by a random matrix theory modeling of critical systems. We trace this quantum-classical discrepancy to strong diffraction effects that dominate the dynamics.

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Systems with a phase transition were always a fruitful subject of study for many areas of theoretical and experimental physics. Specifically in the field of random media, the celebrated Anderson metal-insulator transition (MIT) [1] has been an exciting subject of research for more than 50 years. On the other hand, quantum chaos brought up a connection between quantized chaotic systems and localization ideas emerging from solid-state physics [2]. It has been shown that quantum suppression of classical diffusion is a result of wave interference phenomena of similar nature as the ones responsible for the Anderson localization in random media. Quite recently the connection between the two fields was further strengthened with the observation that certain non-Kolmogorov-Arnold-Moser dynamical systems exhibiting classically anomalous diffusion can have statistical properties resembling the ones of media at MIT [3,4]. This phenomenon is referred to as *critical chaos*. Some of these properties include, quantum anomalous diffusion [5], multifractal wave functions [6], critical spectral [7], and delay time statistics [8]. Many of these properties can be exactly derived using nonconventional ensembles of random matrices with variance decaying from the diagonal in a power-law fashion [1,3,9], which in turn model a variety of systems [10].

Most of the above studies discuss the stationary properties of critical systems. On the other hand this knowledge is often not sufficient for a complete description of the dynamics. This need led us in recent years to focus on new measures that efficiently probe the complexity of quantum time evolution. One such measure is the so-called Loschmidt echo (LE) which probes the sensitivity of quantum dynamics to external perturbations (for recent reviews see [11]). The recent literature on the subject is quite vast and ranges in areas as diverse as atomic optics [12–14], microwaves [15], elastic waves [16], quantum information [17], and quantum chaos [18–23,25,26]. Formally, the LE, $F(t)$, is defined as

$$F(t) = |\langle \psi_0 | e^{iH_0 t} e^{-iH_\epsilon t} | \psi_0 \rangle|^2, \quad \hbar = 1, \quad (1)$$

where $H_\epsilon = H_0 + \epsilon W$ is a one-parameter family of the Hamiltonians, H_0 is the unperturbed Hamiltonian, W represents a perturbation of strength ϵ , and $|\psi_0\rangle$ is an initial state.

For a *noncritical* quantum system with a chaotic classical

counterpart, the decay of the LE depends on the strength of the perturbation parameter ϵ . Three regimes have been identified: the standard perturbative, the Fermi golden rule, and the nonperturbative regime. The first two can be described by linear-response theory leading to a decay which depends on the perturbation strength ϵ as $F(t) \sim e^{-(\epsilon t)^2}$ and $F(t) \sim e^{-\epsilon^2 t}$, respectively [19,22]. In the nonperturbative regime, the LE initially follows a Lyapunov decay $F(t) \sim e^{-\lambda t}$, with a rate given by the Lyapunov exponent λ of the underlying classical system [18,19], whereas for longer times (beyond the so-called Ehrenfest time) the LE decays in accordance with the classical autocorrelation function [23]. This behavior matches the decay of the classical echo according to the correspondence principle.

In this Rapid Communication we make the first step in understanding the echo decay of dynamical systems with critical chaos. The focus of the presentation is on the nonperturbative regime where our study revealed a result (see Fig. 1): we have discovered the appearance of an *echo gap* (initial drop of LE) $1 - F_g$ for initial states which are distributed in the parts of phase-space where the Hamiltonian function exhibits singularities. We have found that at the shortest classical time scale t^* (mean free time between singular scattering events) $F_g = F(t^*)$ scales as

$$F_g(\alpha, \epsilon, \eta) = \begin{cases} f_{cl}(\chi_{cl}) & \text{where } \chi_{cl} = C \frac{\eta^{3-\alpha}}{\epsilon} \\ f_q(\chi_q) & \text{where } \chi_q = C \frac{\eta^2}{\epsilon}, \end{cases} \quad (2)$$

where the subindices q/cl indicate the quantum/classical scaling function for F_g . In Eq. (2), α is the order of singularity of a nonanalytical potential, η is the characteristic spread of the phase-space density of the initial classical or quantum state (e.g., its Wigner function), and the constant C only depends on details of H_0 [24]. Moreover, we found that the scaling function $f(\chi)$ behaves asymptotically as $f(\chi \rightarrow 0) \sim \chi$. The above scaling laws were derived based on the analysis of classical dynamics and confirmed nicely by numerical simulations.

The apparent deviation of the quantum echo behavior

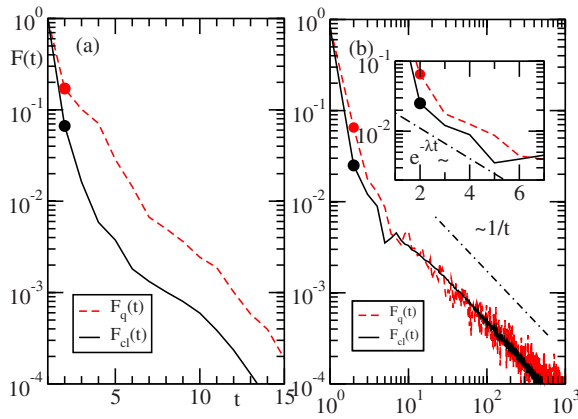


FIG. 1. (Color online) Quantum (red dashed line) and classical (black solid line) LE $F(t)$ for the KR with $V(q)=\log|q|$ and $K_0=1$ in the nonperturbative regime: (a) torus geometry with $L=1$, $N=2^{17}$, and classical perturbation $\epsilon=10^{-4}$ (corresponding to $\sigma\approx 2.0$). The width of the initial preparation is $\eta\approx 0.04275$. (b) Cylinder geometry, with $L=10^3$, $N=2^{16}$, and classical perturbation $\epsilon=0.4$ (corresponding to $\sigma\approx 4.17$). One observes that after the initial Lyapunov decay (inset; the dashed line indicates an average Lyapunov decay), $F(t)$ follows a power law decay given by the autocorrelation function [4,23]. Here we have $\eta\approx 0.15$. In both cases we have used more than 10^7 trajectories for the classical calculation, while an averaging over 800 initial Gaussian wave packets centered at different momenta p_0 and $q_0=0$ has been done in the quantum calculation. The filled circles indicate the first (classical or quantum) nontrivial time step of $F_g=F(n=2)$. The statistical errors are smaller than the symbol size.

from its classical counterpart [see Eq. (2)] can be seen as a violation of the quantum-classical correspondence (QCC); the latter being confirmed in all previous fidelity studies. We have found that the origin of this anomalous behavior is due to strong diffraction effects which dictate the wave dynamics for the class of dynamical systems we investigate in this Rapid Communication.

Below we consider a class of parametric kicked rotors (KR) defined by the time-dependent Hamiltonian [4]

$$H_0 = \frac{p^2}{2} + K_0 V(q) \sum_n \delta(t - nT), \quad (3)$$

where (p, q) is a pair of canonical variables and T and K_0 are the period and the strength of the kicking potential, respectively. The class of KRs that we will study below has a potential which is given by

$$V(q) = \begin{cases} |q|^\alpha & \text{for } \alpha \neq 0 \\ \log|q| & \text{for } \alpha = 0. \end{cases} \quad (4)$$

The $\alpha=0$ case corresponds to MIT [4] and will be investigated below in detail. The classical dynamics is described by the following map:

$$p_{n+1} = p_n - K_0 V'(q_n), \quad q_{n+1} = q_n + T p_{n+1}, \quad (5)$$

where all variables are calculated immediately after one map iteration and $V' \equiv \frac{\partial V(q)}{\partial q}$. The domain of q is within the interval $-\pi < q < \pi$. The map [Eq. (5)] can be

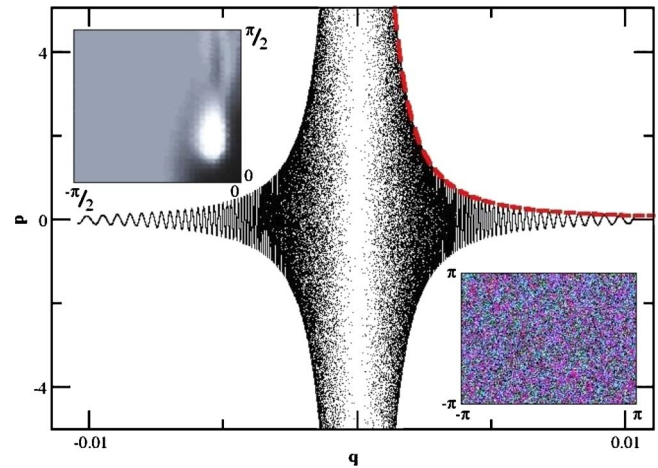


FIG. 2. (Color online) Classical echo map [Eq. (8)] for the $\alpha=0, K_0=1$ singular potential (cylinder geometry with $L=10^3$). The initial preparation is a box of size $\eta=\pi/300$ around $(0,0)$. The evolution snapshot is for the shortest nontrivial time scale $n=2$. The thick dashed (red) line is the theoretical prediction of Eq. (9). Upper left inset: The Wigner function representation (torus geometry) of the quantum echo map. The light areas correspond to negative phase-space densities indicating diffraction phenomena. Lower right inset: a typical phase space for $K_0=1$ (20 trajectories involved up to time 10^5 iterations of the map).

studied on a cylinder $p \in (-\infty, \infty)$, which can also be closed to form a torus of length $2\pi L$, where L is an integer. For $K_0 > 0$ the motion is chaotic (see inset of Fig. 2) with a (local) Lyapunov exponent given by $\lambda(q) = 2 \log\{1 + K_0/(2q^2) + \sqrt{K_0/q^2 + [K_0/(2q^2)]^2}\}$ for $q \neq 0$.

The quantum evolution is described by a one-step unitary operator \hat{U}_0 acting on the wave function $\psi(q)$:

$$U_0 = \exp(-i\hat{n}^2/2)\exp[-ikV(q)], \quad \hbar = 1, \quad (6)$$

where $\hat{n} = -i\partial/\partial q$, $-N/2 \leq n \leq N/2$, $\tau = (2\pi L/N)T$, and $k = (N/2\pi L)K_0$. Optionally, we define an effective Planck constant $\hbar_{\text{eff}} = 2\pi L/N$. The classical limit corresponds to $N \rightarrow \infty$. Without loss of generality we will assume below that $T=1$. It was shown in [4] that for $-0.5 \leq \alpha \leq 0.5$ the eigenfunctions of the above unitary operator are multifractal while the levels resemble the statistical properties of disordered systems at MIT.

For the echo calculation, we perturbed our system with the following (smooth) potential $W(q) = \cos q$. Correspondingly the perturbed quantum kicking parameter is $\sigma = \epsilon/\hbar_{\text{eff}}$. Quantum mechanically, the initial preparation is a Gaussian wave packet centered along the line of singularity, i.e., $(q_0, p_0) = (0, p_0)$. The linear width 2η of the packet is taken to be minimal (i.e., $2\eta = \Delta p = \Delta q = \sqrt{\hbar_{\text{eff}}/2}$), where we perform an averaging over different p_0 's in order to eliminate fluctuations. The corresponding classical initial preparation is given by a uniform distribution of trajectories located inside a box of area $A = 2\eta \times 2\eta \sim \hbar_{\text{eff}}$. We then define the classical LE, $F_{\text{cl}}(t=n)$, as the overlap of the initial area A_0 with the area \tilde{A}_f obtained by evolving A_0 for n iterations of the perturbed map and then reversing the evolution for n iterations with the

unperturbed one. We have also checked that the results remain qualitatively the same when we chose an initial classical distribution to be a Gaussian density, equivalent to a Wigner function of the quantum Gaussian wave packet.

We start our analysis with the classical derivation of Eq. (2). To this end we consider the classical echo dynamics [27]. We denote by $\Phi_0(p, q)$ the forward symplectic map defined in Eq. (5), while by $\Phi_\epsilon = \Phi_0 \circ P_\epsilon$ we denote the corresponding perturbed forward map. In this notation $P_\epsilon(p, q) = (p + \epsilon \sin(q), q)$ is a symplectic map generated by the perturbation (perturbation map). In this framework, the n -step echo map is defined as

$$\Phi_n^E \equiv \Phi_0^{-n} \circ \Phi_\epsilon^n = \tilde{P}_\epsilon^{(n-1)} \circ \tilde{P}_\epsilon^{(n-1)} \circ \dots \circ \tilde{P}_\epsilon^{(0)}, \quad (7)$$

where the perturbation map is written in the interaction picture, i.e., $\tilde{P}_\epsilon^{(n)} \equiv \Phi_0^{-n} \circ P_\epsilon \circ \Phi_0^n$. Explicitly, $(p_{n+1}^E, q_{n+1}^E) = \tilde{P}_\epsilon^{(n)}(p_n^E, q_n^E)$, where

$$p_{n+1}^E = p_n^E + \epsilon \sin[\Phi_0^n(p_n^E, q_n^E)]_q \frac{\partial[\Phi_0^n(p_n^E, q_n^E)]_q}{\partial q_n^E},$$

$$q_{n+1}^E = q_n^E - \epsilon \sin[\Phi_0^n(p_n^E, q_n^E)]_q \frac{\partial[\Phi_0^n(p_n^E, q_n^E)]_q}{\partial p_n^E}. \quad (8)$$

For singular potentials and initial conditions close to singularity, we express the phase-space shift produced by the echo map after the second iteration step:

$$\Delta p_2^E = p_2^E - p_0^E \approx -\epsilon \sin(q_1) K_0 V''(q_0),$$

$$\Delta q_2^E = q_2^E - q_0^E \approx -\epsilon \sin(q_1), \quad (9)$$

where q_1 is given by the map [Eq. (5)] with the initial condition $(p_0^E, q_0^E) = (p_0, q_0)$. Due to the fact that the initial conditions are populating a box centered on the singularity line, we can assume that $\sin(q_1)$ is a pseudorandom variable with density $\mathcal{P}[x \equiv \sin(q)] = (1/\pi)(1-x^2)^{-1/2}$. The accuracy of our assumption is tested in Fig. 2, where we compare the envelope of the shift in momentum as it is given by Eq. (9) with the exact echo dynamics.

For a given q_0 (and assuming $\epsilon \ll \eta$), the probability to return to the initial phase-space box is estimated as

$$\mathcal{P}(q_0) = \frac{1}{\pi} \frac{\eta}{\eta + \epsilon K_0 |V''(q_0)|}, \quad (10)$$

provided that the typical echo shift Δp_2 is much larger than η . The echo probability is just the integral of $\mathcal{P}(q_0)$ over the initial interval $q_0 \in [-\eta, \eta]$:

$$F_{\text{cl}}^g = \frac{1}{2\eta} \int_{-\eta}^{\eta} \mathcal{P}(q_0) dq_0 \approx \frac{1}{2\pi\epsilon K_0} \int_{-\eta}^{\eta} \frac{dq_0}{|V''(q_0)|}. \quad (11)$$

For the specific family of singular potentials discussed in this Rapid Communication, the above relation gives us

$$F_{\text{cl}}^g = \begin{cases} \frac{1}{3\pi K_0} \frac{\eta^3}{\epsilon} & \text{for } \alpha = 0 \\ \frac{1}{\pi K_0 |\alpha(\alpha-1)|(3-\alpha)} \frac{\eta^{3-\alpha}}{\epsilon} & \text{for } \alpha \neq 0. \end{cases} \quad (12)$$

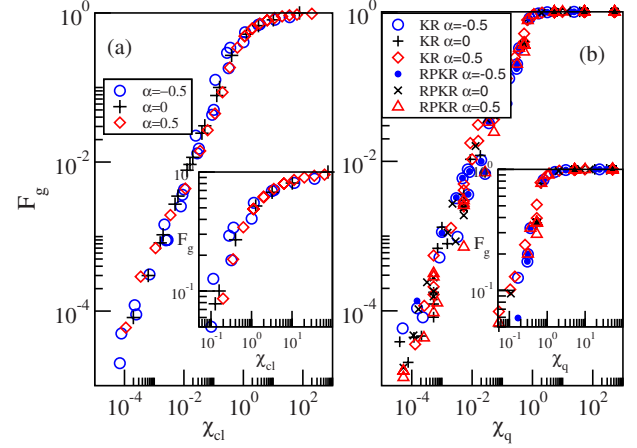


FIG. 3. (Color online) The F_g for various α, η, ϵ 's of the KR defined by Eqs. (3) and (4). In (a) we report the classical F_g against the scaling variable χ_{cl} [see Eq. (2)], while in (b) we report the corresponding quantum F_g vs the scaling variable χ_q [see Eq. (2)]. At the same subfigure we report the results for the RPKR resulting from the model [Eq. (6)] by a randomization of the phases of the kinetic part of U_0 . In the insets we report a magnification of the main panels in the regime of $F_g \approx 1$. In all cases an excellent data collapse is observed.

Our results in Eq. (12) are nicely confirmed in Fig. 3, where we are plotting the echo gap for various α, ϵ and η values by making use of the rescaled variable χ_{cl} given by Eq. (2). Strictly speaking, the above results are applicable only for the case where $F_{\text{cl}}^g \ll 1$. Nevertheless, our numerics indicate that the scaling behavior Eq. (2) continues to apply for values of $F_{\text{cl}}^g \sim 1$.

We have also tested the results of the classical analysis against the quantum echo gap. A complete breakdown of the QCC is observed after the shortest nontrivial time scale (two iteration steps). This is associated with the fact that the Ehrenfest time for our system $t_E \sim \log(h_{\text{eff}})/\lambda(q_0) \rightarrow 0$ when $q_0 \rightarrow 0$ [28]. Weak correspondence is restored for longer times when the echo dynamics spreads ergodically resulting in a vanishing measure of the critical line at $q_0 = 0$.

In Fig. 3 we observe that although the ϵ dependence of the quantum F_g is captured by the classical calculations, both the $\eta \sim \sqrt{\hbar}$ and the α dependence differ drastically. The latter can be explained by a random phase kicked rotor (RPKR) with singular potential, which is simply given by Eq. (6) and replacing the eigenvalues of $\hat{m}^2/2$ by random phases. This indicates that the appearance of a gap is insensitive to classical dynamics and thus can be captured by a random matrix theory (RMT) modeling which preserves the power-law band structure of the evolution operator. Thus the fidelity gap is a universal phenomenon of critical systems described by these RMT models and can be used as an alternative criterium to level or wave-function statistics [1,4-7].

We argue that the violation of the QCC is due to dominating diffraction effects that appear as a consequence of the singular potential. This is illustrated in the inset of Fig. 2 where we show the Wigner function (computed according to Ref. [29]) of the echo map for the first nontrivial time step. One observes the appearance of nonclassical regions in the

phase space where the Wigner function takes negative values.

In conclusion, we find that the LE of dynamical systems exhibiting critical chaos decays instantaneously with a gap that scales inverse proportionally to the strength of the perturbation. The order of the potential singularity is encoded in the scaling properties of the classical echo gap, while the corresponding quantum gap is insensitive to it and its scaling

properties are described by an RMT modeling of critical systems. This deviation is explained in the basis of strong diffraction which is a dominating mechanism of short-time echo dynamics.

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